

Theory of an optical dipole trap for cold atoms

B. M. Garraway

*Sussex Centre for Optical and Atomic Physics, School of Chemistry, Physics, and Environmental Sciences, University of Sussex,
Falmer, Brighton BN1 9QJ, England*

V. G. Minogin

Institute of Spectroscopy, Russian Academy of Sciences, 142092, Troitsk, Moscow Region, Russia

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The theory of an atom dipole trap composed of a focused, far red-detuned, trapping laser beam, and a pair of red-detuned, counterpropagating, cooling beams is developed for the simplest realistic multilevel dipole interaction scheme based on a model of a $(3+5)$ -level atom. The description of atomic motion in the trap is based on the quantum kinetic equations for the atomic density matrix and the reduced quasiclassical kinetic equation for atomic distribution function. It is shown that when the detuning of the trapping field is much larger than the detuning of the cooling field, and with low saturation, the one-photon absorption (emission) processes responsible for the trapping potential can be well separated from the two-photon processes responsible for sub-Doppler cooling atoms in the trap. Two conditions are derived that are necessary and sufficient for stable atomic trapping. The conditions show that stable atomic trapping in the optical dipole trap can be achieved when the trapping field has no effect on the two-photon cooling process and when the cooling field does not change the structure of the trapping potential but changes only the numerical value of the trapping potential well. It is concluded that the separation of the trapping and cooling processes in a pure optical dipole trap allows one to cool trapped atoms down to a minimum temperature close to the recoil temperature, keeping simultaneously a deep potential well.

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I. INTRODUCTION

In recent years there has been a growing interest in developing traps for cold atoms with applications to fundamental physical problems, nanotechnology, high-resolution microwave and optical spectroscopy, frequency standards and atom optics [1–5]. Among different types of atom traps studied in recent years, of practical importance is a far-off-resonance optical dipole trap (FORT) based on a single focused red-detuned laser beam [6–10]. The FORT produces a nearly conservative potential well for atoms, but incorporates the inevitable heating due to the photon recoil associated with the scattered laser light [8,9]. Although the heating rate may be very small at very large detuning from the resonance, the photon recoil heating inevitably introduces an upper limit on the lifetime of atoms in the trap.

It was previously proposed that the heating mechanism in the FORT might be suppressed by adding the cooling laser fields to a focused trapping laser beam [7,8]. Latest experiments with different types of cooling laser fields have shown that the addition of the cooling field can increase the lifetime of atoms and even the atomic density in the trap [11,12].

The addition of the cooling field changes not only the Boltzmann factor that defines the lifetime of atoms in the FORT, but may also have a profound effect on all the basic parameters of the trap since the cooling field may strongly influence the atomic populations and coherences. Typically, any cooling laser field operates at a detuning less than the detuning of the trapping laser beam. The cooling field may thus be responsible not only for perturbation of the steady-state internal atomic state but the perturbation of the trapping potential as well. Up to now, the FORT with an additional

cooling laser field was theoretically discussed for the simplest model of a two-level dipole interaction scheme [7,13]. A two-level model has, however, a very limited connection with real experimental techniques that typically explore multilevel dipole interaction schemes. Physically, there is a major difference between the models of a two-level atom and a multilevel atom in applications related to the trapping and cooling atoms. In a two-level model, both trapping and cooling fields excite atoms on the same atomic transition. As a result, for a two-level atomic scheme in the FORT, the depth of the potential well and the cooling limit have generally the same order of magnitude [7,13] defined by a so-called Doppler temperature [14]. In multilevel atomic schemes, the trapping and cooling laser fields can produce principally different atomic transitions. The trapping field can produce a potential well due to the one-photon transitions while the cooling laser field can cool atoms down to the sub-Doppler temperatures [15–18] due to the two-photon transitions [19,20]. It is thus important that in multilevel atomic schemes the optical processes used for trapping atoms in the FORT and those for sub-Doppler cooling can have the different physical origin. The use of the different optical processes for trapping and cooling multilevel atoms thus raises new questions on basic parameters of the trap and the lifetime of atoms achievable in the FORT with a superimposed sub-Doppler cooling process.

In this paper, we present a theoretical analysis of the FORT composed of a single red-detuned trapping laser beam and a red-detuned cooling field composed of the counterpropagating circular-polarized laser waves. To be specific, we develop an analytical theory for a simplest realistic model of a $(3+5)$ -level atom interacting with one-dimensional

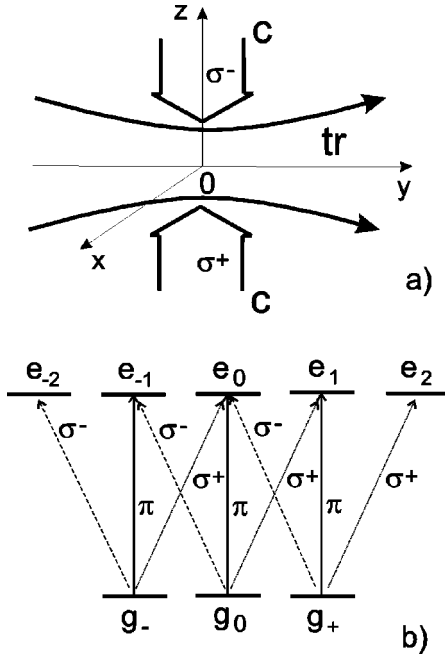


FIG. 1. One-dimensional scheme of the FORT composed of a red-detuned trapping laser beam (tr) and the red-detuned cooling laser field (c) in the $\sigma^+ - \sigma^-$ configuration (a) and the dipole interaction scheme of a (3+5)-level atom with the trapping and cooling laser fields (b). The magnetic sublevels for the ground and excited states of the atom are denoted as: $|j_g = 1, m\rangle = g_m$, $|j_e = 2, m\rangle = e_m$.

counterpropagating laser waves in a $\sigma^+ - \sigma^-$ configuration. In this model, the dipole potential is caused by the one-photon processes, while the friction force comes from the two-photon processes.

Our analysis shows that under specific conditions formulated below the one-photon processes, producing the optical dipole potential can have a negligible influence on the two-photon processes responsible for sub-Doppler cooling of atoms. On the other hand, the perturbation of the atomic internal state produced by the two-photon cooling processes may not change the structure of the potential well but only the value of the potential well depth. The separation of the trapping and cooling processes achievable under specified conditions allows one to maintain the depth of the potential well in the FORT that exceeds considerably the kinetic energy of trapped atoms. As a result, the lifetime of atoms associated with the photon scattering in the FORT can be extremely long, thus leaving other processes, such as atom collisions and the reabsorption of laser light [21–24], to be responsible for the atom storage time.

II. MULTILEVEL INTERACTION SCHEME

The simplest scheme for a FORT may be described by the dipole interaction of a (3+5)-level atom with a linearly polarized focused trapping laser beam and one-dimensional cooling $\sigma^+ - \sigma^-$ laser-field configuration as shown in Fig. 1. In the scheme, a far red-detuned laser beam is assumed to be responsible for atomic trapping due to the one-photon optical

processes while a less red-detuned $\sigma^+ - \sigma^-$ laser field is responsible for laser cooling due to the two-photon optical processes. In the chosen interaction scheme, the field of a linearly polarized trapping laser beam

$$\mathbf{E}_t = \mathbf{e}_0 E_{0t}(\mathbf{r}) \cos(ky - \omega_t t) \quad (1)$$

is defined by a unit polarization vector $\mathbf{e}_0 = \mathbf{e}_z$, spatially non-uniform amplitude $E_{0t}(\mathbf{r})$ and the wave vector $\mathbf{k}_t = k\mathbf{e}_y$ ($k \approx \omega_t/c$). The cooling laser field is assumed to be composed of two counterpropagating left circularly polarized waves,

$$\begin{aligned} \mathbf{E}_c = & \frac{1}{2} E_{0c} (\mathbf{e}_+ e^{i(kz - \omega_c t)} - \mathbf{e}_- e^{-i(kz - \omega_c t)}) \\ & + \frac{1}{2} E_{0c} (-\mathbf{e}_+ e^{i(kz + \omega_c t)} + \mathbf{e}_- e^{-i(kz + \omega_c t)}), \end{aligned} \quad (2)$$

where $\mathbf{e}_{\pm} = \mp (1/\sqrt{2})(\mathbf{e}_x \pm i\mathbf{e}_y)$ are spherical unit vectors, ω_c is the frequency of the cooling laser waves, and $k \approx \omega_c/c$ is the magnitude of the wave vector. With respect to the quantization axis Oz , the first wave in Eq. (2) is a σ^+ polarized wave and the second one is a σ^- polarized wave. Note that in a nonrelativistic approach the magnitude of the wave vector for the counterpropagating cooling laser fields is considered to be the same.

For the above interaction scheme, the atomic Hamiltonian has a standard form

$$H = H_0 - \frac{\hbar^2}{2M} \nabla^2 - \mathbf{D} \cdot \mathbf{E}, \quad (3)$$

where the Hamiltonian H_0 describes the internal atomic states with energy levels $E_{g_0}, E_{g_{\pm}}$, and $E_{e_0}, E_{e_{\pm}}, E_{e_{\pm 2}}$, and the last term describes the dipole interaction between the atom and the total laser field $\mathbf{E} = \mathbf{E}_t + \mathbf{E}_c$.

III. UNPERTURBED TRAPPING POTENTIAL

Consider first the trapping potential for the above interaction scheme in the absence of the cooling field. The quasi-classical atomic-density-matrix equations describing the dipole interaction of a (3+5)-level atom with the field (1) in a rotating wave approximation (RWA) are listed in Appendix A. The set includes only equations for the density matrix elements that do not decay to zero for an interaction time $t \gg \gamma^{-1}$. The density matrix elements entering the equations of Appendix A are defined with respect to the time-dependent atomic wave functions. The equations include, on the left-hand side, the total (convective) time derivatives

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}}. \quad (4)$$

The Rabi frequency for the trapping laser beam, Ω , and the detuning, Δ , are:

$$\Omega = \Omega(\mathbf{r}) = \frac{\|d\| E_{0t}(\mathbf{r})}{\sqrt{30}\hbar}, \quad \Delta = \omega_t - \omega_0, \quad (5)$$

where $\|d\|$ is the reduced dipole matrix element and ω_0 is the atomic transition frequency. The Rabi frequency is defined with respect to the strongest π -type dipole transition $|j_g = 1, m_g = 0\rangle - |j_e = 2, m_e = 0\rangle$. The spontaneous decay rate W_{sp} is defined in the usual way as,

$$W_{\text{sp}} = 2\gamma = \frac{4}{3} \frac{\|d\|^2 \omega_0^3}{\hbar c^3}. \quad (6)$$

Eliminating an explicit time and position dependence in the equations for the density matrix elements with the substitutions

$$\begin{aligned} \rho_{g-e-} &= \sigma_{g-e-} e^{-i(ky - \Delta t)}, & \rho_{g_0 e_0} &= \sigma_{g_0 e_0} e^{-i(ky - \Delta t)}, \\ \rho_{g+e+} &= \sigma_{g+e+} e^{-i(ky - \Delta t)}, \end{aligned}$$

and solving next the equations for a steady state, one can find the dipole radiation force on a (3+5)-level atom with the use of a well-justified quasiclassical formula

$$\mathbf{F} = \nabla \langle \langle \mathbf{D} \rangle \cdot \mathbf{E} \rangle, \quad (7)$$

where $\langle \mathbf{D} \rangle = D_{ab} \rho_{ba}$ is an expectation value of the atomic dipole moment and the gradient ∇ is assumed to act on the electric field \mathbf{E} only. The radiation force on a (3+5)-level atom in a laser beam (1) includes, as usual, two partial forces, i.e., the gradient force and the radiation pressure force,

$$\mathbf{F} = \mathbf{F}_{\text{gr}} + \mathbf{F}_{\text{rp}}, \quad (8)$$

$$\mathbf{F}_{\text{gr}} = 2\hbar [\nabla \Omega(\mathbf{r})] \text{Re} \left(\sigma_{g_0 e_0} + \sqrt{\frac{3}{2}} (\sigma_{g-e-} + \sigma_{g+e+}) \right), \quad (9)$$

$$\mathbf{F}_{\text{rp}} = 2\hbar k \mathbf{e}_y \text{Im} \left(\sigma_{e_0 g_0} + \sqrt{\frac{3}{2}} (\sigma_{e-g-} + \sigma_{e+g+}) \right). \quad (10)$$

The case of large negative detuning is of particular interest in this paper, i.e., we will let $|\Delta| \gg \Omega(\mathbf{r})$, and for a motionless atom the forces are then

$$\mathbf{F}_{\text{gr}} = \frac{30}{17} \hbar \frac{\nabla \Omega^2(\mathbf{r})}{|\Delta|}, \quad (11)$$

$$\mathbf{F}_{\text{rp}} = \frac{30}{17} \hbar k \gamma \mathbf{e}_y \frac{\Omega^2(\mathbf{r})}{\Delta^2}. \quad (12)$$

The gradient force (11) produces the potential well for cold atoms that differs from that for a two-level atom [7] by a numerical factor only, that is,

$$U = - \int \mathbf{F}_{\text{gr}} \cdot d\mathbf{r} = - \frac{30}{17} \hbar \frac{\Omega^2(\mathbf{r})}{|\Delta|}. \quad (13)$$

The radiation pressure force (12) produces an additional asymmetric potential well that can be neglected at large detuning.

IV. QUANTUM KINETIC EQUATIONS

In general, to describe the atomic motion, we can use the approach based on the quantum kinetic equations for the atomic density matrix in the Wigner representation. This is relatively straightforward for a scheme such as the one considered in Fig. 1, where the atom interacts with both laser fields (1) and (2). Since we wish to analyze the operation of the trap at large detunings, it is sufficient for our purpose to consider the quantum kinetic equations at small optical saturation. We will take into account one- and two-photon optical processes, and neglect higher-order optical processes. To simplify the consideration of the equations of motion, we write out the initial microscopic equations, but we neglect first the spatial variation in the trapping laser beam amplitude $E_{0t}(\mathbf{r})$. The effect of the spatially inhomogeneous trapping laser beam will be taken into account in Sec. VI. A set of the atomic density matrix equations in the Wigner representation and a rotating wave approximation (RWA) describing an interaction of a (3+5)-level atom with laser waves of constant amplitude at weak optical saturation is listed in Appendix B. The set only includes the equations for the density matrix elements describing one- and two-photon processes. As before, the density matrix elements entering the equations of Appendix B are defined with respect to the time-dependent atomic wave functions. The upper indices for the momentum-shifted density matrix elements are chosen to have different meaning for the terms coming from the interaction with a trapping and cooling field, i.e.,

$$\Omega \rho_{ab}^{(\pm)} = \Omega \langle a | \rho(\mathbf{r}, \mathbf{p} \pm \frac{1}{2} \hbar k \mathbf{e}_y, t) | b \rangle,$$

$$\theta \rho_{ab}^{(\pm)} = \theta \langle a | \rho(\mathbf{r}, \mathbf{p} \pm \frac{1}{2} \hbar k \mathbf{e}_z, t) | b \rangle, \quad (14)$$

$$\rho_{ab}^{(\mathbf{n})} = \langle a | \rho(\mathbf{r}, \mathbf{p} + \mathbf{n} \hbar k, t) | b \rangle,$$

where \mathbf{n} is a unit vector that defines the direction of the spontaneous photon emission. The total time derivative on the left-hand side of the equations is defined as before by Eq. (4). The Rabi frequency for the cooling laser field, θ (defined with respect to the strongest σ -type dipole transition) and the detuning of the cooling field are

$$\theta = \frac{\|d\| E_{0c}}{2\sqrt{5}\hbar}, \quad \delta = \omega_c - \omega_0. \quad (15)$$

Functions $\Phi_\lambda(\mathbf{n})$, with $\lambda = \sigma, \pi$, define the angular anisotropy of the spontaneous photon emission,

$$\Phi_\sigma(\mathbf{n}) = \frac{3}{16\pi} (1 + n_z^2), \quad \Phi_\pi(\mathbf{n}) = \frac{3}{8\pi} (1 - n_z^2), \quad (16)$$

where n_z is the projection of the unit vector \mathbf{n} on the quantization axis Oz . Numerical factors entering the density matrix equations are due to the Clebsch-Gordan coefficients defining the dipole matrix elements for specific atomic transitions. Our choice of sign of the reduced dipole moment can be seen from the values of the matrix elements for the strongest π -type and σ -type atomic transitions,

$$\begin{aligned}
\langle e_0 | d^0 | g_0 \rangle &= \langle g_0 | d^0 | e_0 \rangle = \sqrt{\frac{2}{15}} \|d\|, \\
\langle e_{-2} | d^- | g_{-} \rangle &= -\langle g_{-} | d^+ | e_{-2} \rangle = \langle e_{+2} | d^+ | g_{+} \rangle \\
&= -\langle g_{+} | d^- | e_{+2} \rangle = \frac{1}{\sqrt{5}} \|d\|.
\end{aligned}$$

Note that when the cooling laser field and photon recoil are neglected, the equations of Appendix B reduce to the quasi-classical equations of Appendix A. The quantum kinetic equations of Appendix B are used below to derive the quasi-classical kinetic equation of atomic motion that defines the dipole radiation force on the atom in the total laser field, the momentum diffusion tensor, and the potential well for the motionless atom.

V. QUASICLASSICAL KINETIC EQUATION

The initial quantum kinetic equations for the atomic density matrix elements can be first simplified by the standard procedure of eliminating the explicit time and position dependence with proper substitutions. Since the initial equations include only terms describing one- and two-photon processes, we introduce substitutions that also take into account only one- and two-photon processes:

$$\begin{aligned}
\rho_{g_{-}e_{-2}} &= \sigma_{g_{-}e_{-2}} e^{ikz+i\delta t}, & \rho_{g_{-}e_{-}} &= \sigma_{g_{-}e_{-}} e^{-iky+i\Delta t}, \\
\rho_{g_{-}e_0} &= \sigma_{g_{-}e_0} e^{-ikz+i\delta t}, \\
\rho_{g_0e_{-}} &= \sigma_{g_0e_{-}} e^{ikz+i\delta t}, & \rho_{g_0e_0} &= \sigma_{g_0e_0} e^{-iky+i\Delta t}, \\
\rho_{g_0e_{+}} &= \sigma_{g_0e_{+}} e^{-ikz+i\delta t}, \\
\rho_{g_{+}e_0} &= \sigma_{g_{+}e_0} e^{ikz+i\delta t}, & \rho_{g_{+}e_{+}} &= \sigma_{g_{+}e_{+}} e^{-iky+i\Delta t}, \\
\rho_{g_{+}e_{+2}} &= \sigma_{g_{+}e_{+2}} e^{-ikz+i\delta t}, \\
\rho_{g_{-}g_0} &= \sigma_{g_{-}g_0}^1 e^{-ik(y+z)+i(\Delta-\delta)t} + \sigma_{g_{-}g_0}^2 e^{ik(y-z)-i(\Delta-\delta)t}, \\
\rho_{g_{-}g_{+}} &= \sigma_{g_{-}g_{+}} e^{-2ikz}, \\
\rho_{g_0g_{+}} &= \sigma_{g_0g_{+}}^1 e^{-ik(y+z)+i(\Delta-\delta)t} + \sigma_{g_0g_{+}}^2 e^{ik(y-z)-i(\Delta-\delta)t}, \\
\rho_{e_{-2}e_{-}} &= \sigma_{e_{-2}e_{-}} e^{-ik(y+z)+i(\Delta-\delta)t}, \\
\rho_{e_{-2}e_0} &= \sigma_{e_{-2}e_0} e^{-2ikz}, \\
\rho_{e_{-}e_0} &= \sigma_{e_{-}e_0}^1 e^{ik(y-z)-i(\Delta-\delta)t} + \sigma_{e_{-}e_0}^2 e^{-ik(y+z)+i(\Delta-\delta)t}, \\
\rho_{e_{-}e_{+}} &= \sigma_{e_{-}e_{+}} e^{-2ikz}, \\
\rho_{e_0e_{+}} &= \sigma_{e_0e_{+}}^1 e^{ik(y-z)-i(\Delta-\delta)t} + \sigma_{e_0e_{+}}^2 e^{-ik(y+z)+i(\Delta-\delta)t}, \\
\rho_{e_0e_{+2}} &= \sigma_{e_0e_{+2}} e^{-2ikz}, & \rho_{e_{+}e_{+2}} &= \sigma_{e_0e_{+2}} e^{ik(y-z)-i(\Delta-\delta)t}.
\end{aligned} \tag{17}$$

Having made the above substitutions, we neglect in the microscopic equations all terms that describe three-photon and higher-order multiphoton processes. This procedure finally results in a set of equations that does not include explicit time and position dependence.

An analysis of the density matrix equations that have no explicit time and coordinate dependence can be done in the usual way [13]. Under the condition that atom-laser field interaction time exceeds the characteristic relaxation times of the atomic density-matrix, the momentum width of the atomic density-matrix elements can be assumed to exceed the photon momentum $\hbar k$. This principal assumption, which will be checked below, allows one to expand the density-matrix elements in powers of the photon momentum $\hbar k$. Considering next the time evolution of the equations expanded in successively increasing orders of the photon momentum $\hbar k$, one can conclude that the diagonal elements ρ_{aa} and the off-diagonal elements σ_{ab} are functionals of the Wigner quasiprobability distribution function $w \equiv w(\mathbf{r}, \mathbf{p}, t)$,

$$\begin{aligned}
w &= \sum \rho_{g_{\alpha}g_{\alpha}} + \sum \rho_{e_{\beta}e_{\beta}}, & \alpha &= -1, 0, 1; \\
\beta &= -2, -1, 0, 1, 2.
\end{aligned} \tag{18}$$

The structure of the functional dependence for the diagonal and off-diagonal density matrix elements can be directly determined from the structure of the expanded equations:

$$\begin{aligned}
\rho_{aa} &= \left(R_{aa}^0 + \frac{1}{2} \hbar k R_{aa}^1 + \dots \right) w + \frac{1}{2} \hbar k (Q_{aa}^1 + \dots) \frac{\partial w}{\partial p_z} + \dots, \\
\sigma_{ab} &= \left(S_{ab}^0 + \frac{1}{2} \hbar k S_{ab}^1 + \dots \right) w + \frac{1}{2} \hbar k (T_{ab}^1 + \dots) \frac{\partial w}{\partial p_z} + \dots,
\end{aligned} \tag{19}$$

where $R_{aa}^0, S_{ab}^0, R_{aa}^1, S_{ab}^1, \dots, Q_{aa}^1, T_{ab}^1, \dots$ are functions of atomic momentum \mathbf{p} , or atomic velocity $\mathbf{v} = \mathbf{p}/M$, that have to be determined by the solution procedure. In accordance with the definition of the quasi-probability distribution function (18), the unknown diagonal functions satisfy the normalization conditions

$$\sum R_{aa}^0 = 1, \quad \sum R_{aa}^1 = 0, \quad \sum Q_{aa}^1 = 0, \quad \dots \tag{20}$$

Substituting next the general solution (19) into the expanded equations for the atomic density-matrix elements, one can see that the Wigner function $w = w(\mathbf{r}, \mathbf{p}, t)$ satisfies an infinite Fokker-Planck equation. To the second order in the photon momentum $\hbar k$ the equation for the quasiprobability distribution function $w = w(\mathbf{r}, \mathbf{p}, t)$ is then described by a second-order Fokker-Planck kinetic equation,

$$\frac{\partial w}{\partial t} + \mathbf{v} \cdot \frac{\partial w}{\partial \mathbf{r}} = - \frac{\partial}{\partial \mathbf{p}} \cdot (\mathbf{F} w) + \sum \frac{\partial^2}{\partial p_i^2} (D_{ii} w), \tag{21}$$

where $i = x, y, z$, \mathbf{F} is the dipole radiation force on the atom, and D_{ii} is the momentum diffusion tensor.

In this section, we consider only homogeneous laser waves where the dipole radiation force has the clear meaning of a radiation pressure force, $\mathbf{F} = \mathbf{F}_{\text{rp}} = \mathbf{e}_z F_{\text{rp}}$. This force and the momentum diffusion tensor entering Eq. (21) are

$$F_{\text{rp}} = 2\hbar k \Omega \text{Im} \left[(S_{e_+g_+}^0 + S_{e_-g_-}^0) + \frac{1}{\sqrt{2}}(S_{e_+g_0}^0 + S_{e_-g_0}^0) + \frac{1}{\sqrt{6}}(S_{e_0g_-}^0 + S_{e_0g_+}^0) \right], \quad (22)$$

$$D_{ii} = \hbar^2 k^2 \gamma \left[\alpha_{ii}^\sigma \left(R_{e_-e_-}^0 + \frac{1}{2}R_{e_-e_-}^0 + \frac{1}{3}R_{e_0e_0}^0 + \frac{1}{2}R_{e_+e_+}^0 + R_{e_+e_+}^0 \right) + \alpha_{ii}^\pi \left(\frac{1}{2}R_{e_-e_-}^0 + \frac{2}{3}R_{e_0e_0}^0 + \frac{1}{2}R_{e_+e_+}^0 \right) \right] + \delta_{iz} \hbar^2 k^2 \Omega \left[\text{Im}(T_{g_+e_+}^1 - T_{e_-g_-}^1) + \sqrt{2} \text{Im}(T_{e_+g_0}^1 - T_{g_0e_-}^1) + \sqrt{\frac{2}{3}} \text{Im}(T_{g_-e_0}^1 - T_{e_0g_+}^1) \right]. \quad (23)$$

In the above equations the coefficients α_{ii}^σ , α_{ii}^π determine the angular anisotropy of the spontaneous photon emission,

$$\alpha_{ii}^\sigma = \int \Phi_\sigma(\mathbf{n}) n_i^2 d^2\mathbf{n}, \quad \alpha_{ii}^\pi = \int \Phi_\pi(\mathbf{n}) n_i^2 d^2\mathbf{n}, \quad (24)$$

$$\alpha_{xx}^\sigma = \alpha_{yy}^\sigma = \frac{3}{10}, \quad \alpha_{zz}^\sigma = \frac{2}{5}; \quad \alpha_{xx}^\pi = \alpha_{yy}^\pi = \frac{2}{5}, \quad \alpha_{zz}^\pi = \frac{1}{5}. \quad (25)$$

The values of the force and momentum diffusion coefficients, F_{rp} and D_{ii} , which govern the time evolution of the quasiclassical distribution function, can be explicitly determined by solving the steady-state equations that follow from the expanded equations for the atomic-density-matrix elements. These are considered separately in zero and first order in photon momentum $\hbar k$. Explicit equations for the quantities F_{rp} and D_{ii} are given in Sec. VIII.

VI. DIPOLE GRADIENT FORCE

We next take into consideration the dipole gradient force \mathbf{F}_{gr} associated with the gradient of the trapping laser beam amplitude $\mathbf{E}_{0t} = \mathbf{E}_{0t}(\mathbf{r})$. The derivation of the gradient force can be done in a way that generalizes the procedure considered in the preceding section. Representing the trapping laser beam amplitude in the form of a Fourier expansion,

$$\mathbf{E}_t(\mathbf{r}) = (2\pi\hbar)^{-3/2} \int \mathbf{E}_{0t}(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}} d^3q, \quad (26)$$

one should introduce into the equations of Appendix B the following substitutions:

$$i\Omega(\mathbf{r})\rho_{ab}(\mathbf{p}) \rightarrow (2\pi\hbar)^{-3/2} \int i\Omega(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}} \rho_{ab}(\mathbf{p} + \frac{1}{2}\hbar\mathbf{q}) d^3q. \quad (27)$$

For a laser beam amplitude that varies in space on a scale that is large compared to the size of the atomic wave packet, we use an expansion of the density-matrix elements to first order in the small momentum $\hbar\mathbf{q}$,

$$\rho_{ab}(\mathbf{p} + \frac{1}{2}\hbar\mathbf{q}) \approx \rho_{ab}(\mathbf{p}) + \frac{1}{2}\hbar\mathbf{q} \cdot \frac{\partial}{\partial\mathbf{p}} \rho_{ab}(\mathbf{p}). \quad (28)$$

This transforms the terms in equations of Appendix B into

$$i\Omega(\mathbf{r})\rho_{ab}(\mathbf{p}) \rightarrow i\Omega(\mathbf{r})\rho_{ab}(\mathbf{p}) + \frac{\hbar}{2} \nabla\Omega(\mathbf{r}) \cdot \frac{\partial}{\partial\mathbf{p}} \rho_{ab}(\mathbf{p}). \quad (29)$$

When the substitutions (29) are introduced into the equations of Appendix B, the Fokker-Planck equation (21) includes, in the above approximation, the total radiation force as a sum of the dipole gradient force and the radiation pressure force (22),

$$\mathbf{F} = \mathbf{F}_{\text{gr}} + \mathbf{F}_{\text{rp}}. \quad (30)$$

The dipole gradient force is then determined by the steady-state optical coherences as

$$\mathbf{F}_{\text{gr}} = 2\hbar [\nabla\Omega(\mathbf{r})] \text{Re} \left(S_{g_0e_0}^0 + \sqrt{\frac{3}{2}}(S_{g_-e_-}^0 + S_{g_+e_+}^0) \right). \quad (31)$$

Note that formal mathematical expression for this part of the total radiation force coincides with that given for the gradient force by Eq. (9). The explicit expressions given by Eqs. (31) and (9) are generally different since the functions S_{ab}^0 describe the interaction of the atom with the total laser field (defined by the equations of Appendix B), while the functions σ_{ab} describe the interaction of the atom with only the trapping laser beam (defined by the equations of Appendix A). We will see in Sec. VIII that Eq. (31) gives the gradient force (41) that differs by a numerical factor from the gradient force (11) given by Eq. (9).

VII. LOW-SATURATION SOLUTION

We will determine the steady-state atomic density matrix elements, which define the partial forces and the momentum diffusion tensor, in a regime characterized by two important limits. That is, we consider large detunings, where

$$|\Delta|, |\delta| \gg \gamma, \quad (32)$$

and we consider low optical saturation,

$$s_t = \frac{\Omega^2}{\Delta^2} \ll 1, \quad s_c = \frac{\theta^2}{\delta^2} \ll 1, \quad (33)$$

when the one-photon and two-photon processes play a dominant role in the time evolution of the atomic-density-matrix

elements. In addition to the above approximations, we restrict our treatment to the case of slowly moving atoms where

$$\eta_y = \frac{kv_y}{\gamma} \ll 1, \quad \eta_z = \frac{kv_z}{\gamma} \ll 1. \quad (34)$$

Under all these conditions the sets of equations for the functions R_{aa}^0 , S_{ab}^0 , and T_{ab}^1 , which follow from the equations of Appendix B expanded to the first order in the photon momentum, have a simple analytical solution. When solving the last equations, one can note that the efficiencies of the two-photon optical processes between the ground-state magnetic sublevels depend crucially on two parameters μ and ν ,

$$\mu = \frac{5}{6} \frac{\theta^2}{\delta^2} \gamma, \quad \nu = \frac{\Omega^2}{\Delta^2} \gamma, \quad (35)$$

which define the frequency widths of the two-photon resonance structures related to the ground-state coherences ρ_{g-g_+} and $\rho_{g-g_0}, \rho_{g_0g_+}$. Physically, the origin of the two-photon frequency widths can be understood in the following way. The laser field connects the ground-state probability amplitudes with the upper-state probability amplitudes through one-photon excitation processes. These one-photon absorption processes perturb the ground-state probability amplitudes and then lead to decay rates for the ground-state coherences of the order of the rates of the one-photon transitions, i.e., of the order of μ and ν . These quantities play accordingly the role of the frequency widths for the two-photon resonance processes related to the ground-state coherences.

Since the two-photon processes induced by the cooling laser field (2) are only important for deep (sub-Doppler) cooling of atoms in the trap [19,20], the physical meaning of parameters μ and ν allows us to introduce two conditions necessary for separating the cooling and trapping processes: i.e.,

$$\nu \ll \mu \ll \gamma. \quad (36)$$

The left inequality guarantees that the trapping field does not influence the two-photon cooling process, while the right inequality is needed for the two-photon friction coefficient to be greater than the one-photon friction coefficient [20].

Under the condition (36), and in lowest order of the small parameters s_t , s_c , η_y and η_z , the low-saturation, low-velocity solutions give the one-photon coherences entering Eq. (31) as

$$S_{g-e-}^0 = \frac{-i\sqrt{3}\Omega/2}{\gamma+i\Delta} N_-, \quad S_{g_0e_0}^0 = \frac{-i\Omega}{\gamma+i\Delta} N_0, \quad (37)$$

$$S_{g+e+}^0 = \frac{-i\sqrt{3}\Omega/2}{\gamma+i\Delta} N_+,$$

where the steady-state ground-state populations $N_- = R_{g-g-}^0$, $N_0 = R_{g_0g_0}^0$ and $N_+ = R_{g+g+}^0$, connected to the ground-state two-photon coherence ρ_{g-g_+} , are

$$N_- = \frac{1}{2\mathcal{D}} \left(\frac{13}{12} \frac{\theta^4}{\delta^2} - \frac{15}{6} \frac{\theta^2}{|\delta|} kv_z + 9k^2 v_z^2 \right),$$

$$N_0 = \frac{2}{\mathcal{D}} \left(\frac{1}{6} \frac{\theta^4}{\delta^2} + k^2 v_z^2 \right), \quad (38)$$

$$N_+ = \frac{1}{2\mathcal{D}} \left(\frac{13}{12} \frac{\theta^4}{\delta^2} + \frac{15}{6} \frac{\theta^2}{|\delta|} kv_z + 9k^2 v_z^2 \right),$$

and the common denominator is

$$\mathcal{D} = \frac{17}{12} \frac{\theta^4}{\delta^2} + 11k^2 v_z^2. \quad (39)$$

The other steady-state coherences and populations entering Eqs. (22) and (23) are defined by similar equations,

$$S_{g-e-2}^0 = \frac{-i\theta}{\gamma+i\delta} N_-, \quad S_{g-e_0}^0 = \frac{-i\theta/\sqrt{6}}{\gamma+i\delta} N_-,$$

$$S_{g_0e-}^0 = \frac{-i\theta/\sqrt{2}}{\gamma+i\Delta} N_0, \dots \quad (40)$$

Qualitatively, the behavior of the ground-state populations at low velocities is defined by the ground-state coherence ρ_{g-g_+} that is induced by the cooling field. The velocity position of the narrow two-photon structures in the ground-state populations can be seen from the energy conservation law. In an atom rest frame the absorption of a photon from one traveling wave, and emission of a photon into the other traveling wave, results in a two-photon transition between ground-state sublevels $|g_- \rangle$, $|g_+ \rangle$ that does not change the atom energy, $(\omega \pm kv) - (\omega \mp kv) = 0$. Energy conservation thus directly shows that the two-photon resonance structures, which are due to the cooling field, are located at zero velocity, $kv = 0$.

VIII. MODIFIED OPTICAL POTENTIAL DEPTH AND KINETIC ENERGY

The atomic coherences that we have derived, Eqs. (37), together with the values of the ground-state populations, give a new value of the gradient force at zero velocity and negative detuning,

$$\mathbf{F}_{\text{gr}} = \frac{55}{68} \hbar \frac{\nabla \Omega^2(\mathbf{r})}{|\Delta|}. \quad (41)$$

In a corresponding way they give a new value of the potential,

$$U(\mathbf{r}) = - \int \mathbf{F}_{\text{gr}} \cdot d\mathbf{r} = - \frac{55}{68} \hbar \Omega(\mathbf{r}) \frac{\Omega(\mathbf{r})}{|\Delta|}, \quad (42)$$

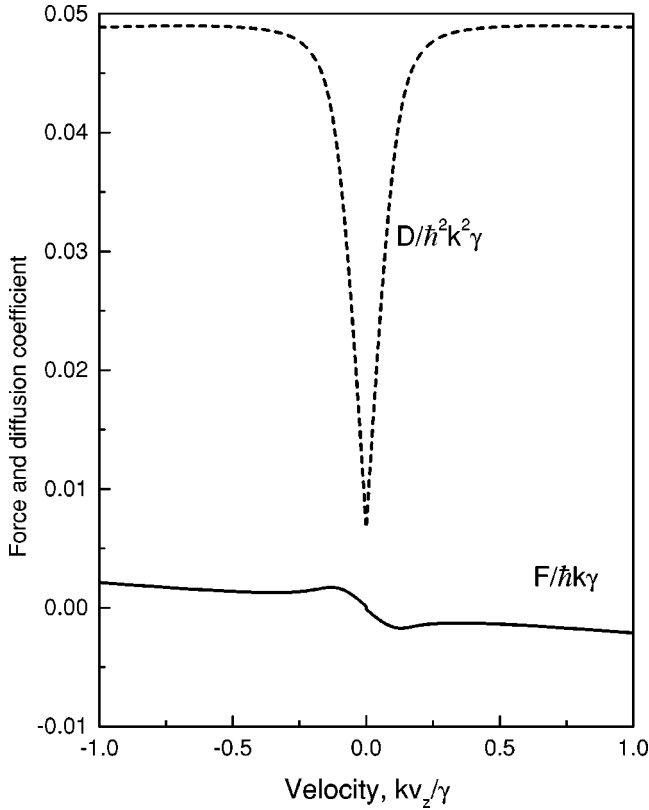


FIG. 2. Velocity dependence of the radiation pressure force F_{rp} (solid line) and diffusion coefficient $D=D_{zz}$ (dashed line) in a small velocity region $|kv_z| < \gamma$ at negative detuning $\delta = -22\gamma$ and saturation parameter $G=5$.

which is about 2.2 times less than the unperturbed potential (13). The reduced value of the gradient force and the potential in the total laser field is naturally explained by the dominant role of the cooling field in producing atomic populations in accordance with the leftmost condition in Eq. (36). While the trapping field alone produces the ground-state atomic populations at zero velocity as $N_- = N_+ = 13/34$, $N_0 = 4/17$, the cooling field redistributes the populations to the values $N_- = N_+ = 9/22$, $N_0 = 4/22$. This redistribution, “multiplied” by the relative strengths of the dipole transitions, gives the decrease in the potential. However, in a three-dimensional cooling field the decrease of the potential should be less profound.

Substitution of the steady-state atomic coherences and ground-state populations into Eq. (22) gives an explicit expression for the radiation pressure force in a low-saturation, low-velocity, approximation. Since the above analytical expressions (38) and (40) neglect a weak velocity dependence due to the one-photon (Doppler) processes, the radiation pressure force shown in Fig. 2 is only described in our analysis near zero velocity, i.e., where $|v| \lesssim \mu/k$. In the case of a red-detuned cooling field, where $\delta < 0$, the low-velocity part of the radiation pressure force reduces to the cooling force. If we represent the force (22) at a red detuning, and at low velocities, in the standard form of a friction force,

$$F_{rp} = -M\beta v, \quad (43)$$

we can derive an explicit expression for the friction coefficient due to the two-photon processes [16]:

$$\beta = \frac{120}{17} \omega_r \frac{\gamma}{|\delta|}, \quad (44)$$

where $\omega_r = \hbar k^2/2M$ is a recoil frequency. The last expression shows that at low effective saturation the friction due to the two-photon processes does not depend on intensity. The two-photon friction is, however, much higher than that due to the one-photon processes. This higher friction is naturally explained by the smaller value of the velocity width of the two-photon structure, μ/k , compared to the width of the one-photon structure, γ/k [see the right inequality (36)].

In a way that is similar to the radiation force, the momentum diffusion tensor D_{ii} also includes a narrow two-photon velocity structure located at zero velocity (Fig. 2). The contribution of the two-photon structure can be shown to decrease the value of the diffusion coefficient $D=D_{zz}$ at zero velocity. The reason is that the analysis of the equations listed in Appendix B shows that the second part of the diffusion coefficient in Eq. (23) is much bigger than the first one. However, this second part of the diffusion coefficient D manifests a narrow velocity dip at velocity $v = v_z = 0$. In our basic approximations, the diffusion coefficient D at zero velocity, D_0 , can be estimated as [20]

$$D_0 = \frac{46}{17} \hbar^2 k^2 \gamma \frac{\theta^2}{\delta^2}. \quad (45)$$

The diffusion coefficient and the friction coefficient jointly define the kinetic energy of the trapped atoms and thus an effective temperature found according to the Einstein relation as

$$E = k_B T = \frac{D_0}{M\beta} = \frac{23}{30} \frac{\hbar \theta^2}{|\delta|}. \quad (46)$$

IX. CONDITIONS FOR STABLE TRAPPING

Assuming now that kinetic energy (46) of cold trapped atoms is much less than the depth $U(0)$ of the potential well (42) one can get a sufficient condition for stable atomic trapping,

$$\frac{\theta^2}{|\delta|} \ll \frac{\Omega^2(0)}{|\Delta|}. \quad (47)$$

Comparing the last condition with the condition for deep laser cooling (36) one can see that both conditions can be satisfied when the detuning of the trapping field is much larger than that of the cooling field,

$$|\Delta| \gg |\delta|. \quad (48)$$

The above two conditions, defined by Eqs. (36) and (47), thus justify the idea of a stable dipole trap. The stable atomic trapping in the optical dipole trap can thus be achieved when the trapping field has no effect on the two-photon cooling process, and when the cooling field does not change the

structure of the trapping potential but changes only the numerical value of the trapping potential well.

The lifetime of atoms in the trap associated with the diffusive heating can finally be estimated as

$$\tau = \varpi e^{U(0)/E}, \quad (49)$$

where ϖ is the oscillation frequency of an atom in the trap and the Boltzmann factor $U(0)/E$ considerably exceeds unity.

The above evaluations of the conditions necessary and sufficient for stable atomic trapping can be illustrated by a specific example. Assume that the Rabi frequency and the detuning for a cooling field are accordingly $\theta = 1\gamma$ and $|\delta| = 20\gamma$ and the Rabi frequency and the detuning for the trapping field are $\Omega = 10^2\gamma$ and $|\Delta| = 10^4\gamma$. For these parameters the one-photon widths $\nu = 10^{-4}\gamma$ and $\mu = 2.5 \times 10^{-3}\gamma$ satisfy conditions (36) for deep laser cooling. In their turn, the kinetic energy (46), estimated as $E = 0.04\hbar\gamma$, and the potential-well depth, estimated according to Eq. (42) as $U(0) = 0.8\hbar\gamma$, satisfy the sufficient condition (47) for stable atomic trapping giving for the Boltzmann factor a sufficiently large value $U(0)/E = 20$.

X. CONCLUSION

The above analysis shows thus that in an optical dipole trap, one-photon processes responsible for trapping atoms can be well separated from the two-photon processes responsible for the deep (sub-Doppler) cooling of atoms. This separation of the trapping and cooling processes ensures, in turn, a stable operation of the trap according to Eq. (49). This principal conclusion is also valid for the three-dimensional case since if one adds new $\sigma^+ - \sigma^-$ cooling fields along the axes Ox and Oy , all the above conclusions remain valid with only the numerical coefficients changing. This justifies the possibility of separating the trapping and cooling processes in a three-dimensional case.

Finally, note that the above quasiclassical analysis cannot give a quantitative limit for the temperature of laser-cooled atoms since it considers atomic velocities higher than the recoil velocity. The cooling limit can, however, be seen from a qualitative estimation. According to the above analysis, the two-photon cooling process can operate when the velocity width of the two-photon resonance exceeds the recoil velocity, $\delta v = \mu/k > v_r$. That is, when the atomic energy exceeds the recoil energy, $E > E_r \approx \hbar^2 k^2 / 2M$. These limits give the physical justification for the key assumption that the atomic-density-matrix elements can be expanded in powers of the photon momentum $\hbar k$.

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APPENDIX A

Below is listed a set of quasiclassical equations for the atomic-density-matrix elements in the RWA that describe the dipole interaction of a (3+5)-level atom with the trapping laser field defined by Eq. (1). Note that the set includes only the equations that involve density-matrix elements not vanishing at the interaction time $t \gg \gamma^{-1}$.

$$\begin{aligned} \frac{d}{dt} \rho_{g-g-} &= \frac{i\sqrt{3}}{2} \Omega e^{-i(ky-\Delta t)} \rho_{e-g-} + \text{c.c.} \\ &+ \gamma(\rho_{e-e-} + \frac{1}{3} \rho_{e_0 e_0}), \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} \rho_{g_0 g_0} &= i\Omega e^{-i(ky-\Delta t)} \rho_{e_0 g_0} + \text{c.c.} \\ &+ \gamma(\rho_{e-e-} + \frac{4}{3} \rho_{e_0 e_0} + \rho_{e+e+}), \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} \rho_{g+g+} &= \frac{i\sqrt{3}}{2} \Omega e^{-i(ky-\Delta t)} \rho_{e+g+} + \text{c.c.} \\ &+ \gamma(\frac{1}{3} \rho_{e_0 e_0} + \rho_{e+e+}), \end{aligned}$$

$$\frac{d}{dt} \rho_{e-e-} = \frac{i\sqrt{3}}{2} \Omega e^{i(ky-\Delta t)} \rho_{g-e-} + \text{c.c.} - 2\gamma \rho_{e-e-},$$

$$\frac{d}{dt} \rho_{e_0 e_0} = i\Omega e^{i(ky-\Delta t)} \rho_{g_0 e_0} + \text{c.c.} - 2\gamma \rho_{e_0 e_0},$$

$$\frac{d}{dt} \rho_{e+e+} = \frac{i\sqrt{3}}{2} \Omega e^{i(ky-\Delta t)} \rho_{g+e+} + \text{c.c.} - 2\gamma \rho_{e+e+},$$

$$\frac{d}{dt} \rho_{g-e-} = -\frac{i\sqrt{3}}{2} \Omega e^{-i(ky-\Delta t)} (\rho_{g-g-} - \rho_{e-e-}) - \gamma \rho_{g-e-},$$

$$\frac{d}{dt} \rho_{g_0 e_0} = -i\Omega e^{-i(ky-\Delta t)} (\rho_{g_0 g_0} - \rho_{e_0 e_0}) - \gamma \rho_{g_0 e_0},$$

$$\frac{d}{dt} \rho_{g+e+} = -i\Omega e^{-i(ky-\Delta t)} (\rho_{g+g+} - \rho_{e+e+}) - \gamma \rho_{g+e+}.$$

APPENDIX B

Below are listed the atomic-density-matrix equations in the Wigner representation, and the RWA. They describe the dipole interaction of a (3+5)-level atom with trapping and cooling laser fields defined by Eqs. (1) and (2). The equations include only terms describing one-photon and two-photon optical processes.

$$\begin{aligned}
\frac{d}{dt}\rho_{g-g-} &= \frac{i\sqrt{3}}{2}\Omega e^{-i(ky-\Delta t)}\rho_{e-g-}^{(+)} + i\theta e^{ikz+i\delta t}\rho_{e-2g-}^{(-)} \\
&\quad + \frac{i}{\sqrt{6}}\theta e^{-ikz+i\delta t}\rho_{e_0g-}^{(+)} + \text{c.c.} \\
&\quad + \gamma \int [2\Phi_{\sigma}(\mathbf{n})\rho_{e-2e-}^{(\mathbf{n})} + \Phi_{\pi}(\mathbf{n})\rho_{e-e-}^{(\mathbf{n})} \\
&\quad + \frac{1}{3}\Phi_{\sigma}(\mathbf{n})\rho_{e_0e_0}^{(\mathbf{n})}]d^2\mathbf{n}, \\
\frac{d}{dt}\rho_{g_0g_0} &= i\Omega e^{-i(ky-\Delta t)}\rho_{e_0g_0}^{(+)} + \frac{i}{\sqrt{2}}\theta e^{ikz+i\delta t}\rho_{e-g_0}^{(-)} \\
&\quad + \frac{i}{\sqrt{2}}\theta e^{-ikz+i\delta t}\rho_{e+g_0}^{(+)} + \text{c.c.} + \gamma \int (\Phi_{\sigma}(\mathbf{n})\rho_{e-e-}^{(\mathbf{n})} \\
&\quad + \frac{4}{3}\Phi_{\pi}(\mathbf{n})\rho_{e_0e_0}^{(\mathbf{n})} + \Phi_{\sigma}(\mathbf{n})\rho_{e+e+}^{(\mathbf{n})})d^2\mathbf{n}, \\
\frac{d}{dt}\rho_{g+g+} &= \frac{i\sqrt{3}}{2}\Omega e^{-i(ky-\Delta t)}\rho_{e+g+}^{(+)} + i\theta e^{-ikz+i\delta t}\rho_{e+2g+}^{(+)} \\
&\quad + \frac{i}{\sqrt{6}}\theta e^{ikz+i\delta t}\rho_{e_0g+}^{(-)} + \text{c.c.} + \gamma \int (\frac{1}{3}\Phi_{\sigma}(\mathbf{n})\rho_{e_0e_0}^{(\mathbf{n})} \\
&\quad + \Phi_{\pi}(\mathbf{n})\rho_{e+e+}^{(\mathbf{n})} + 2\Phi_{\sigma}(\mathbf{n})\rho_{e+2e+}^{(\mathbf{n})})d^2\mathbf{n}, \\
\frac{d}{dt}\rho_{e-2e-2} &= i\theta e^{-ikz-i\delta t}\rho_{g-e-2}^{(+)} + \text{c.c.} - 2\gamma\rho_{e-2e-2}, \\
\frac{d}{dt}\rho_{e-e-} &= \frac{i\sqrt{3}}{2}\Omega e^{i(ky-\Delta t)}\rho_{g-e-}^{(-)} + \frac{i}{\sqrt{2}}\theta e^{-ikz-i\delta t}\rho_{g_0e-}^{(+)} + \text{c.c.} \\
&\quad - 2\gamma\rho_{e-e-}, \\
\frac{d}{dt}\rho_{e_0e_0} &= i\Omega e^{i(ky-\Delta t)}\rho_{g_0e_0}^{(-)} + \frac{i}{\sqrt{6}}\theta(e^{ikz-i\delta t}\rho_{g-e_0}^{(-)} \\
&\quad + e^{-ikz-i\delta t}\rho_{g+e_0}^{(+)}) + \text{c.c.} - 2\gamma\rho_{e_0e_0}, \\
\frac{d}{dt}\rho_{e+e+} &= \frac{i\sqrt{3}}{2}\Omega e^{i(ky-\Delta t)}\rho_{g+e+}^{(-)} + \frac{i}{\sqrt{2}}\theta e^{ikz-i\delta t}\rho_{g_0e+}^{(-)} + \text{c.c.} \\
&\quad - 2\gamma\rho_{e+e+}, \\
\frac{d}{dt}\rho_{e+2e+2} &= i\theta e^{ikz-i\delta t}\rho_{g+e+2}^{(-)} + \text{c.c.} - 2\gamma\rho_{e+2e+2},
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dt}\rho_{g-e-2} &= \frac{i\sqrt{3}}{2}\Omega e^{-i(ky-\Delta t)}\rho_{e-e-2}^{(+)} + i\theta(e^{ikz+i\delta t}\rho_{e-2e-2}^{(-)} \\
&\quad - e^{ikz+i\delta t}\rho_{g-g-}^{(+)}) + \frac{i}{\sqrt{6}}\theta e^{-ikz+i\delta t}\rho_{e_0e-2}^{(+)} \\
&\quad - \gamma\rho_{g-e-2}, \\
\frac{d}{dt}\rho_{g-e-} &= -\frac{i\sqrt{3}}{2}\Omega e^{-i(ky-\Delta t)}(\rho_{g-g-}^{(-)} - \rho_{e-e-}^{(+)}) \\
&\quad + i\theta e^{ikz+i\delta t}\rho_{e-2e-}^{(-)} + \frac{i}{\sqrt{6}}\theta e^{-ikz+i\delta t}\rho_{e_0e-}^{(+)} \\
&\quad - \frac{i}{\sqrt{2}}\theta e^{ikz+i\delta t}\rho_{g-g_0}^{(+)} - \gamma\rho_{g-e-}, \\
\frac{d}{dt}\rho_{g-e_0} &= -i\Omega e^{-i(ky-\Delta t)}\left(\rho_{g-g_0}^{(-)} - \frac{\sqrt{3}}{2}\rho_{e-e_0}^{(+)}\right) \\
&\quad + i\theta e^{ikz+i\delta t}\rho_{e-2e_0}^{(-)} - \frac{i}{\sqrt{6}}\theta(e^{-ikz+i\delta t}\rho_{g-g-}^{(-)} \\
&\quad + e^{ikz+i\delta t}\rho_{g-g+}^{(+)} - e^{-ikz+i\delta t}\rho_{e_0e_0}^{(+)}) - \gamma\rho_{g-e_0}, \\
\frac{d}{dt}\rho_{g_0e-} &= -\frac{i\sqrt{3}}{2}\Omega e^{-i(ky-\Delta t)}(\rho_{g_0g-}^{(-)} - \rho_{e_0e-}^{(+)}) \\
&\quad - \frac{i}{\sqrt{2}}\theta(e^{ikz+i\delta t}\rho_{g_0g_0}^{(-)} - e^{ikz+i\delta t}\rho_{e-e-}^{(-)} \\
&\quad - e^{-ikz+i\delta t}\rho_{e+e-}^{(+)}) - \gamma\rho_{g_0e-}, \\
\frac{d}{dt}\rho_{g_0e_0} &= -i\Omega e^{-i(ky-\Delta t)}(\rho_{g_0g_0}^{(-)} - \rho_{e_0e_0}^{(+)}) \\
&\quad + \frac{i}{\sqrt{2}}\theta(e^{ikz+i\delta t}\rho_{e-e_0}^{(-)} + e^{-ikz+i\delta t}\rho_{e+e_0}^{(+)}) \\
&\quad - \frac{i}{\sqrt{6}}\theta(e^{-ikz+i\delta t}\rho_{g_0g-}^{(-)} + e^{ikz+i\delta t}\rho_{g_0g+}^{(+)}) - \gamma\rho_{g_0e_0}, \\
\frac{d}{dt}\rho_{g_0e+} &= -\frac{i\sqrt{3}}{2}\Omega e^{-i(ky-\Delta t)}(\rho_{g_0g+}^{(-)} - \rho_{e_0e+}^{(+)}) \\
&\quad - \frac{i}{\sqrt{2}}\theta(e^{-ikz+i\delta t}\rho_{g_0g_0}^{(-)} - e^{-ikz+i\delta t}\rho_{e+e+}^{(+)}) \\
&\quad - e^{ikz+i\delta t}\rho_{e-e+}^{(-)}) - \gamma\rho_{g_0e+},
\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\rho_{g+e_0} = & -i\Omega e^{-i(ky-\Delta t)}\left(\rho_{g+g_0}^{(-)} - \frac{\sqrt{3}}{2}\rho_{e_+e_0}^{(+)}\right) \\ & + i\theta e^{-ikz+i\delta t}\rho_{e_+2e_0}^{(+)} - \frac{i}{\sqrt{6}}\theta(e^{ikz+i\delta t}\rho_{g+g_+}^{(+)} \\ & + e^{-ikz+i\delta t}\rho_{g+g_-}^{(-)} - e^{ikz+i\delta t}\rho_{e_0e_0}^{(-)}) - \gamma\rho_{g+e_0},\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\rho_{g+e_+} = & -\frac{i\sqrt{3}}{2}\Omega e^{-i(ky-\Delta t)}(\rho_{g+g_+}^{(-)} - \rho_{e_+e_+}^{(+)}) \\ & + \frac{i}{\sqrt{6}}\theta e^{ikz+i\delta t}\rho_{e_0e_+}^{(-)} + i\theta e^{-ikz+i\delta t}\rho_{e_+2e_+}^{(+)} \\ & - \frac{i}{\sqrt{2}}\theta e^{-ikz+i\delta t}\rho_{g+g_0}^{(-)} - \gamma\rho_{g+e_+},\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\rho_{g+e_+2} = & \frac{i\sqrt{3}}{2}\Omega e^{-i(ky-\Delta t)}\rho_{e_+e_+2}^{(+)} - i\theta(e^{-ikz+i\delta t}\rho_{g+g_+}^{(-)} \\ & - e^{-ikz+i\delta t}\rho_{e_+2e_+2}^{(+)}) + \frac{i}{\sqrt{6}}\theta e^{ikz+i\delta t}\rho_{e_0e_+2}^{(-)} \\ & - \gamma\rho_{g+e_+2},\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\rho_{g-g_0} = & i\Omega\left(\frac{\sqrt{3}}{2}e^{-i(ky-\Delta t)}\rho_{e_-g_0}^{(+)} - e^{i(ky-\Delta t)}\rho_{g-e_0}^{(+)}\right) \\ & + \frac{i}{\sqrt{6}}\theta e^{-ikz}(e^{i\delta t}\rho_{e_0g_0}^{(+)} - e^{-i\delta t}\rho_{g-e_-}^{(-)}) \\ & + \gamma\int\left(\sqrt{2}\Phi_\sigma(\mathbf{n})\rho_{e_-2e_-}^{(\mathbf{n})} + \sqrt{\frac{2}{3}}\Phi_\pi(\mathbf{n})\rho_{e_-e_0}^{(\mathbf{n})} \right. \\ & \left. + \frac{1}{\sqrt{3}}\Phi_\sigma(\mathbf{n})\rho_{e_0e_+}^{(\mathbf{n})}\right)d^2\mathbf{n},\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\rho_{g-g_+} = & \frac{i}{\sqrt{6}}\theta(e^{-ikz+i\delta t}\rho_{e_0g_+}^{(+)} - e^{-ikz-i\delta t}\rho_{g-e_0}^{(-)}) \\ & + \gamma\int\left[\sqrt{\frac{2}{3}}\Phi_\sigma(\mathbf{n})\rho_{e_-2e_0}^{(\mathbf{n})} + \Phi_\pi(\mathbf{n})\rho_{e_-e_+}^{(\mathbf{n})} \right. \\ & \left. + \sqrt{\frac{2}{3}}\Phi_\sigma(\mathbf{n})\rho_{e_0e_+2}^{(\mathbf{n})}\right]d^2\mathbf{n},\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\rho_{g_0g_+} = & i\Omega\left(e^{-i(ky-\Delta t)}\rho_{e_0g_+}^{(+)} - \frac{\sqrt{3}}{2}e^{i(ky-\Delta t)}\rho_{g_0e_+}^{(+)}\right) \\ & + \frac{i}{\sqrt{6}}\theta e^{-ikz}(e^{i\delta t}\rho_{e_+g_+}^{(+)} - e^{-i\delta t}\rho_{g_0e_0}^{(-)}) \\ & + \gamma\int\left(\frac{1}{\sqrt{3}}\Phi_\sigma(\mathbf{n})\rho_{e_-e_0}^{(\mathbf{n})} + \sqrt{\frac{2}{3}}\Phi_\pi(\mathbf{n})\rho_{e_0e_+}^{(\mathbf{n})} \right. \\ & \left. + \Phi_\sigma(\mathbf{n})\rho_{e_+e_+2}^{(\mathbf{n})}\right)d^2\mathbf{n},\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\rho_{e_-2e_-} = & -\frac{i\sqrt{3}}{2}\Omega e^{-i(ky-\Delta t)}\rho_{e_-2g_-}^{(-)} + i\theta e^{-ikz-i\delta t}\rho_{g_-e_-}^{(+)} \\ & - 2\gamma\rho_{e_-2e_-},\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\rho_{e_-2e_0} = & i\theta e^{-ikz-i\delta t}\rho_{g_-e_0}^{(+)} - \frac{i}{\sqrt{6}}\theta e^{-ikz+i\delta t}\rho_{e_-2g_-}^{(-)} \\ & - 2\gamma\rho_{e_-2e_0},\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\rho_{e_-e_0} = & i\Omega\left(\frac{\sqrt{3}}{2}e^{i(ky-\Delta t)}\rho_{g_-e_0}^{(-)} - e^{-i(ky-\Delta t)}\rho_{e_-g_0}^{(-)}\right) \\ & + \frac{i}{\sqrt{2}}\theta e^{-ikz-i\delta t}\rho_{g_0e_0}^{(+)} - \frac{i}{\sqrt{6}}\theta e^{-ikz+i\delta t}\rho_{e_-g_-}^{(-)} \\ & - 2\gamma\rho_{e_-e_0},\end{aligned}$$

$$\frac{d}{dt}\rho_{e_-e_+} = \frac{i}{\sqrt{2}}\theta e^{-ikz}(e^{-i\delta t}\rho_{g_0e_+}^{(+)} - e^{i\delta t}\rho_{e_-g_0}^{(-)}) - 2\gamma\rho_{e_-e_+},$$

$$\begin{aligned}\frac{d}{dt}\rho_{e_0e_+} = & i\Omega\left(e^{i(ky-\Delta t)}\rho_{g_0e_+}^{(-)} - \frac{\sqrt{3}}{2}e^{-i(ky-\Delta t)}\rho_{e_0g_+}^{(-)}\right) \\ & - \frac{i}{\sqrt{2}}\theta e^{-ikz+i\delta t}\rho_{e_0g_0}^{(-)} + \frac{i}{\sqrt{6}}\theta e^{-ikz-i\delta t}\rho_{g_+e_+}^{(+)} \\ & - 2\gamma\rho_{e_0e_+},\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\rho_{e_0e_+2} = & -i\theta e^{-ikz+i\delta t}\rho_{e_0g_+}^{(-)} + \frac{i}{\sqrt{6}}\theta e^{-ikz-i\delta t}\rho_{g_+e_+2}^{(+)} \\ & - 2\gamma\rho_{e_0e_+2},\end{aligned}$$

$$\begin{aligned}\frac{d}{dt}\rho_{e_+e_+2} = & \frac{i\sqrt{3}}{2}\Omega e^{i(ky-\Delta t)}\rho_{g_+e_+2}^{(-)} - i\theta e^{-ikz+i\delta t}\rho_{e_+g_+}^{(-)} \\ & - 2\gamma\rho_{e_+e_+2}.\end{aligned}$$

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